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Journal of Structural Geology 27 (2005) 475–479

JOURNAL OF STRUCTURAL GEOLOGY

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Finite strain analysis using ammonoids: an interactive approach

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Received 9 February 2004; received in revised form 30 October 2004; accepted 8 November 2004
Available online 12 January 2005

Abstract

The implementation of computer-based finite strain methodologies not only speeds up the estimation procedures, but usually also increases their accuracy. In this paper an interactive procedure is presented, where the user is able to rapidly fit standard deformed logarithmic spirals to natural examples.

Keywords: Strain analysis; Ammonoids; Logarithmic spiral

1. Introduction

Similar geometrical forms can be obtained by different deformation mechanisms. So the shape of geological structures, such as folds, is often insufficient when trying to understand the structural processes (e.g. Ramsay, 1967, p. 343; Price and Cosgrove, 1990, pp. 249–250; Hudleston and Lan, 1993). Finite strain analysis is a powerful tool in helping to constrain the possible solutions (e.g. Twiss and Moores, 1992, p. 314). Mainly after the classical paper of Cloos (1947), a diversity of methodologies has been proposed in order to estimate finite strain in tectonites. Using mostly the distortion of objects (e.g. ooids and fossils) or point distributions (e.g. quartz grain centers in quartzites) all methods try to estimate the shape and orientation of the strain ellipse/ellipsoid. Although in the early work, finite strain techniques were limited by the calculations involved, the introduction of computer techniques soon led to improvements of the most common strain methodologies. The R_f/ϕ method (Ramsay, 1967, pp. 202–211; Dunnet, 1969) was greatly enhanced by adding the possibility of applying statistical tests in the comparison of deformed fabric data with computed theoretical standard curves (Lisle, 1985). The Fry method (Fry, 1979) was updated to provide a better discrimination of the strain ellipse when studying fabrics in poorly sorted rocks (Erslev, 1988).

However, for some of the less used methodologies, only the classical approach continues to be available. Strain estimation using deformed spiral form (e.g. ammonoids) is such a case. Indeed, although existing methodologies were developed under the assumption that these shells have grown according to a logarithmic spiral law (Mosely, 1838; Thompson, 1942) the mathematical difficulties of working with the entire spiral led to methods using only discrete data points (Blake, 1878; Tan, 1973).

In this paper we present a computer-based methodology in which the finite strain of distorted spiral forms is estimated using the global shape of a set of discrete measurements.

2. Strain analysis using ammonoids

Although the possibility of using ammonoids as strain markers was first envisaged more than one hundred years ago (Blake, 1878), applications have been very limited. Indeed, the inability to deal with three dimensions, the stratigraphical limitations and the predominance of forms with the plane of symmetry parallel to the bedding plane, strongly limits the use of ammonoids in strain studies. Nevertheless, ammonoids can be useful in order to study strain heterogeneities at a regional scale (Subieta, 1977) or

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doi:10.1016/j.jsg.2004.11.002

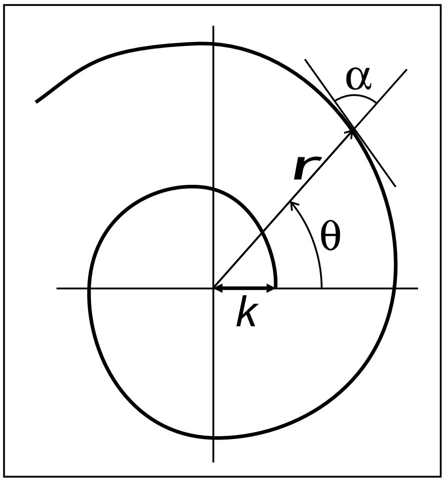

Fig. 1. Logarithmic spiral geometrical parameters.

for constraining the amount of diagenetic compaction (Rocha and Dias, 2003).

2.1. Geometrical assumptions

The recognition that the development of ammonoid shells should obey a logarithmic rule (Mosely, 1838; Thompson, 1942) opened the way for the establishment of several methods to estimate the strain from deformed specimens. So, the subsequent methodologies have all been based on the same mathematical expression for the undeformed logarithmic spiral:

$$r = ke^{\theta \cot \alpha} \quad (1)$$

where r = radius of the spiral at angle θ, k = scale parameter, and α = angle between the tangent to the spiral and the line connecting the tangent point to the spiral center (coiling angle of mollusc).

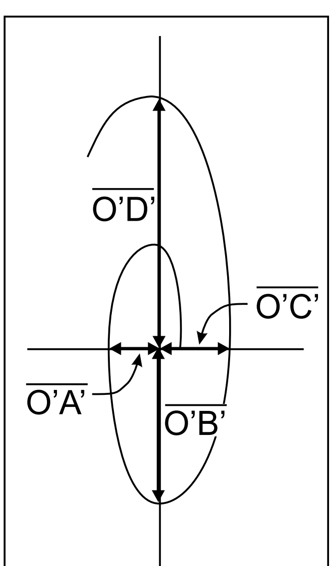

Fig. 2. Parameters used in Blake's method.

This formula (Fig. 1) shows that only one parameter, the spiral angle (α), controls the diversity of shape in ammonoid shells. Tan (1973) stated that this angle always ranges between 80 and 84° for the Jurassic ammonoids he studied.

2.2. Previous methods

In order to estimate the strain from a deformed ammonoid, the theoretical deformed logarithmic spiral that best fits the shape of the fossil shell should be found. Due to the complexity of the calculations involved in the fitting of two spirals, the theoretical one and the natural one, previous methods (Blake, 1878; Tan, 1973) chose a different approach. Instead of working with the complete shape of the deformed fossil, measurements were taken at discrete places along the curved fossil shell. From these data, not only the axial strain ratio of the deformed logarithmic spiral, but also the characteristic spiral angle could be found. Nevertheless, both classical methods, either using linear measurements (Fig. 2; Blake, 1878) or angular ones (Fig. 3; Tan, 1973), never compared the total shape of the deformed fossil with the theoretical curves. This limits the possibility for the user to get a visual idea of the misfit between the estimated spiral and the deformed ammonoid. Tan's methodology partially overcomes this limitation, because fossil data are plotted on graphs on which are also plotted theoretical curves for selected spiral angles (α) and different strain ratios (R; Fig. 4A). The curve that best fits the data is chosen to provide the best estimation of strain. In real situations (Fig. 4B) the fossil data always have some scattering in relation to the theoretical curves. This could happen either because the ammonoid shells did not grow obeying a logarithmic rule, or because of errors in estimating the geometrical parameters, mainly because it is usually difficult to find the correct position of the shell center. Some mathematical best-fit technique (e.g. using polynomials) could then be used in order to find the best curve. Although this process gives some idea of the deviation between the shell and the logarithmic curve, it continues to lack a way for the user to visualize the error in the spiral adjustment. This is a major concern because field geologists often have reason to question the application of pure mathematical procedures (De Paor, 1990).

Also, for estimating the orientation of the finite strain ellipse, the two classical methodologies take opposite approaches. While Tan's procedure uses geometrical measures that are independent of any assumption about the strain ellipse orientation, Blake's older procedure involves choosing by visual inspection of the deformed shell the orientation of the principal strain axis. This is not a simple task:

– Firstly, because the major strain axis does not contain the points of maximum and minimum curvature of the deformed spiral (Fig. 5).

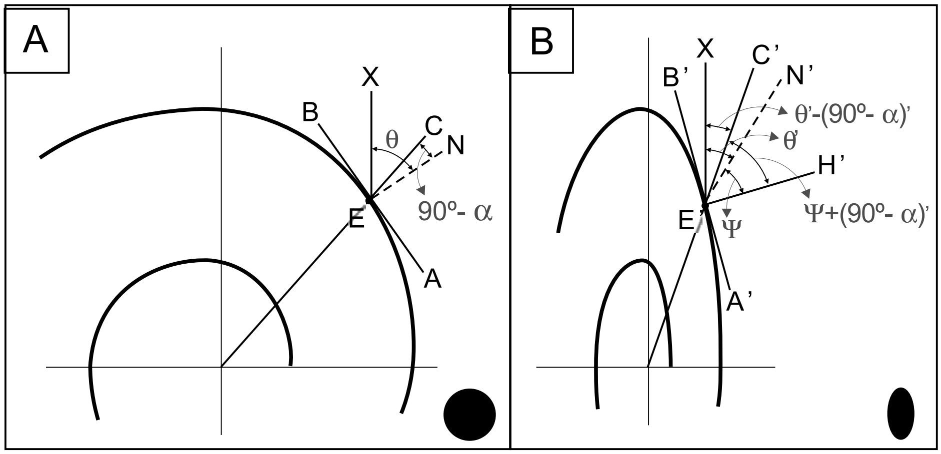

Fig. 3. (A) Tan's parameters in (A) undeformed logarithmic spiral; (B) deformed logarithmic spiral.

– Secondly, due to the difficulty in accurately identifying the points of maximum and minimum curvature in the deformed shell. Although these points do not coincide with the major strain axis of the logarithmic spiral, they still constitute the best visual approximation to it.

The inaccuracy involved in identifying the major strain axis orientation induces errors in the estimation of the strain axial ratios (Fig. 6). For low to moderate strains, errors hardly exceed 5%. However, for longer deformations they could easily attain 10%.

Finally, Blake's (1878) and Tan's (1973) approaches are also not user-friendly because they are time-consuming, especially the latter.

2.3. A global approach

Traditional approaches of strain analysis do not allow the comparison of deformed ammonoids with particular

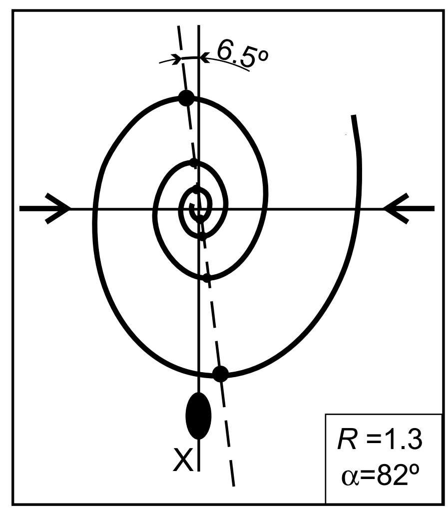

Fig. 5. Misfit between the major strain axis and the line joining points at maximum curvature in a deformed logarithmical spiral with a coiling angle (α) of 82° and an axial strain ratio (R) of 1.3.

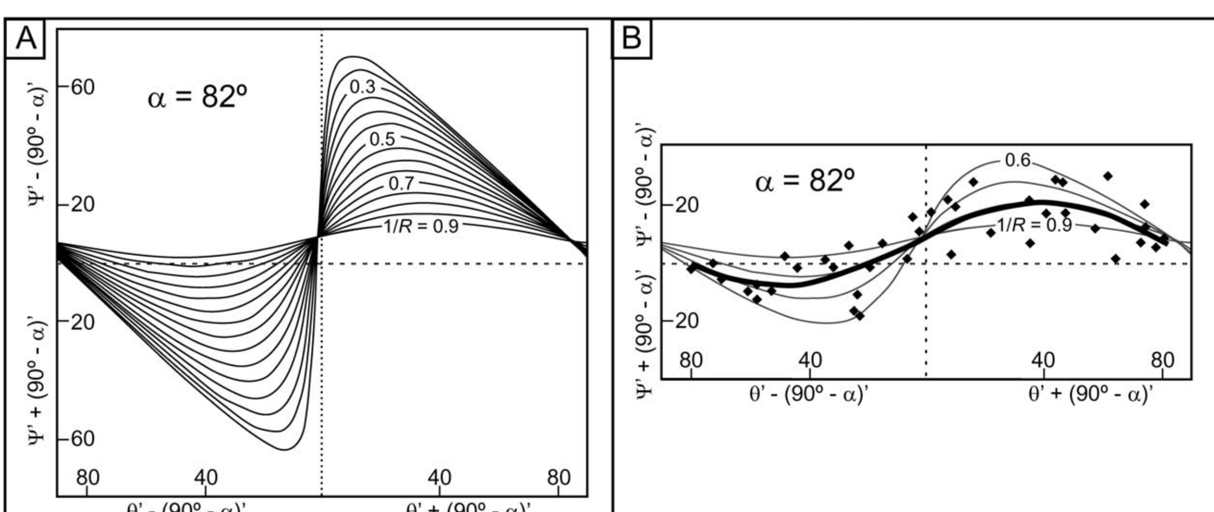

Fig. 4. (A) Standard curves of Tan's method for different strain ratios (R) and a fixed coiling angle (α) of 82°. (B) Real example of fitting discrete data points representing a deformed Jurassic ammonoid (*Asthenoceras* sp.) to a standard curve using Tan's method (Rocha and Dias, 2003).

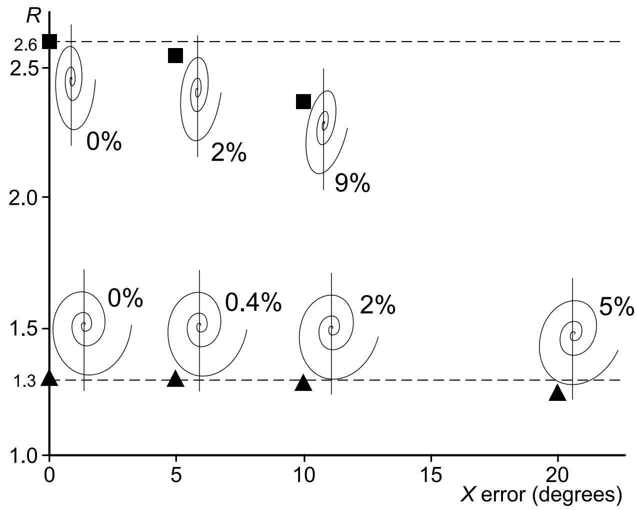

Fig. 6. Errors induced using Blake's method due to the inaccuracy in identifying the major strain axis, for low strain (R=1.3) and high strain (R=2.6).

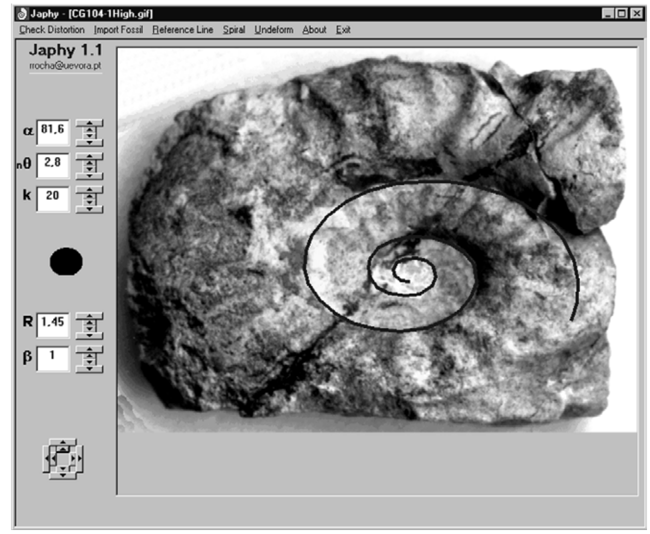

Fig. 8. Display of the interactive computer base technique used in the strain estimation of a deformed ammonoid (*Asthenoceras* sp.).

deformed logarithmic spirals. The parameters of the spiral (Eq. (1)) are complex enough to make such a manual procedure impractical. An alternative approach is to automate the construction of deformed logarithmic spirals. This is the chosen approach.

The computation of the coordinates of points belonging to a deformed logarithmic spiral (Fig. 7A) is a straightforward procedure (Ramsay and Huber, 1983, p. 140):

$$x' = ike^{\theta\cot\alpha}R^{0.5}\cos\theta \tag{2}$$

$$y' = ke^{\theta\cot\alpha}R^{-0.5}\sin\theta \tag{3}$$

where x' and y' = Cartesian coordinates of points in the spiral,

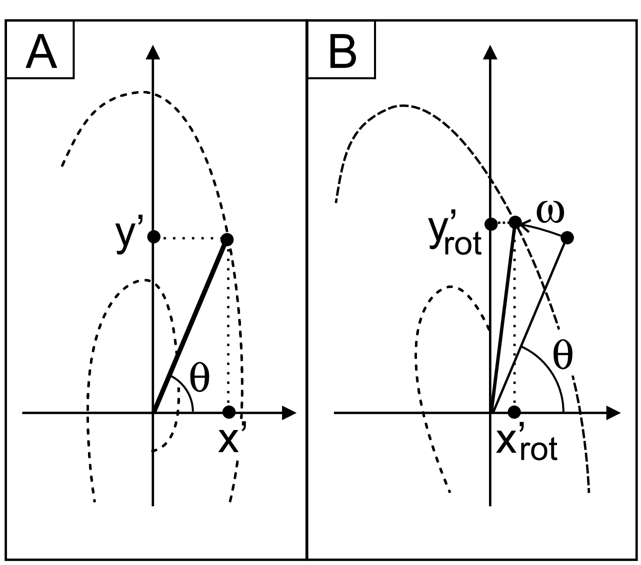

Fig. 7. (A) Parameters used for the deformation of a logarithmic spiral. (B) Parameters used for the rotation of a logarithmic spiral.

i = 1 or −1, respectively, for anti-clockwise or clockwise coiling, and R = axial strain ratio.

The above formulation produces a spiral stretched along the Y-axis. The rotation of this curve (Fig. 7B) can be easily obtained using:

$$x'_{\mathrm{rot}} = \cos(\theta+\omega)((x')^2+(y')^2)^{0.5} \tag{4}$$

$$y'_{\mathrm{rot}} = \sin(\theta+\omega)((x')^2+(y')^2)^{0.5} \tag{5}$$

In the new approach we propose, the spiral defined by Eqs. (2)–(5) can be easily drawn using software (in Rocha, 2003). The user can then overlay this deformed spiral on a previously digitized picture of the fossil. By applying an interactive interface, the user can freely change the variables R, α, k and i of the spiral until he succeeds in finding the best fit. When the adjustment is finished, the parameters of the deformed ammonoid (strain ratio and ellipse orientation) have been obtained.

The advantage of this methodology could be emphasized by its application to real examples. In the Jurassic rocks of central Portugal, an *Asthenoceras* sp. ammonoid deformed during diagenetic compaction has been found (Rocha and Dias, 2003). The use of the refereed software shows a perfect adjustment to a deformed logarithmic spiral with a coiling angle of 81.6° and an axial strain ratio of 1.45 (Fig. 8). The obtained correlation contrasts with the scattering data obtained using Tan's method for the same fossil (Fig. 4B); as previously stated, in such a situation, only the use of specialized mathematical best-fit techniques allows the estimation of the deformation parameters.

However, in some cases the fossil could not be matched to a theoretical deformed logarithmic spiral. This situation could be induced by either heterogeneous deformation, or an initial form not ascribed to a perfect logarithmic spiral or change in growth characteristics of the shell. As expected, the

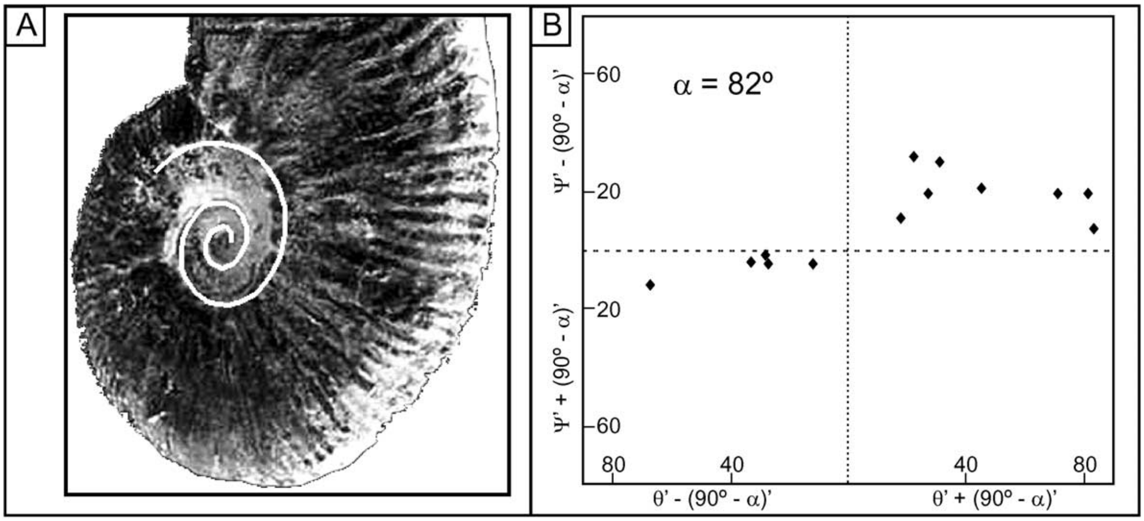

Fig. 9. (A) Misfit of a theoretical logarithmic spiral in a deformed ammonoid (*Kosmoceras grossouvrei*; in Poirot, 2004). (B) Data points using Tan's method in the ammonoid in (A).

use of Tan's methodology in such anomalous forms (Fig. 9A) gives a scattering of the punctual measures in the standard graphics (Fig. 9B), the fit of a significative curve to the data is difficult. However, the proposed computer based technique allows the user to have a clear idea of the misfit between the theoretical spiral and the natural form; a mean value could than be assumed.

3. Conclusions

The fitting of standard deformed spirals to natural strained ammonoid shells is a fast and reliable method. Indeed, although a visual match is still needed, the whole shape of the studied object is used, instead of a set of discrete points. This enables the user to have a clear idea about the eventual misfit of the actual shape and an ideal deformed logarithmic spiral, and thus the degree of confidence that can be placed in the estimated strain parameters and the assumptions inherent in this as in all finite strain methodologies.

Acknowledgements

This is a work of the TEKTONIKOS (Experimental Tectonic and Microtectonic Laboratory of the Évora University). A thorough review by P. Hudleston and R. Lisle greatly helped clarify some of the issues contained herein.

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